Fermi at Trinity

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Abstract

Enrico Fermi estimated the yield of the Trinity test to be about 10 kilotons by dropping small pieces of paper and observing their motion in the blast wave. This is about half the radiochemically derived value of approximately 21 kilotons that necessarily includes thermal and nuclear radiation that do not contribute to the blast. Although this story is classic, there appears to be no account of how he related his observation to the yield. This note attempts to reconstruct how he might have done so.

I. INTRODUCTION

The official history of Los Alamos[1] and many popular accounts describe how Fermi, at Trinity, the first nuclear test, conducted July 16, 1945, estimated its explosive yield with a simple experiment. He dropped some small scraps of paper before the blast wave passed by and observed how far they were displaced. From this, he was able to estimate its explosive yield as 10 kt. This is about 40% of the modern estimate[2], but some portion of that was radiated and did not contribute to the blast wave. Fermi did remarkably well with a very rudimentary experiment, but neither he nor subsequent writers appear to have explained how he inferred the yield.

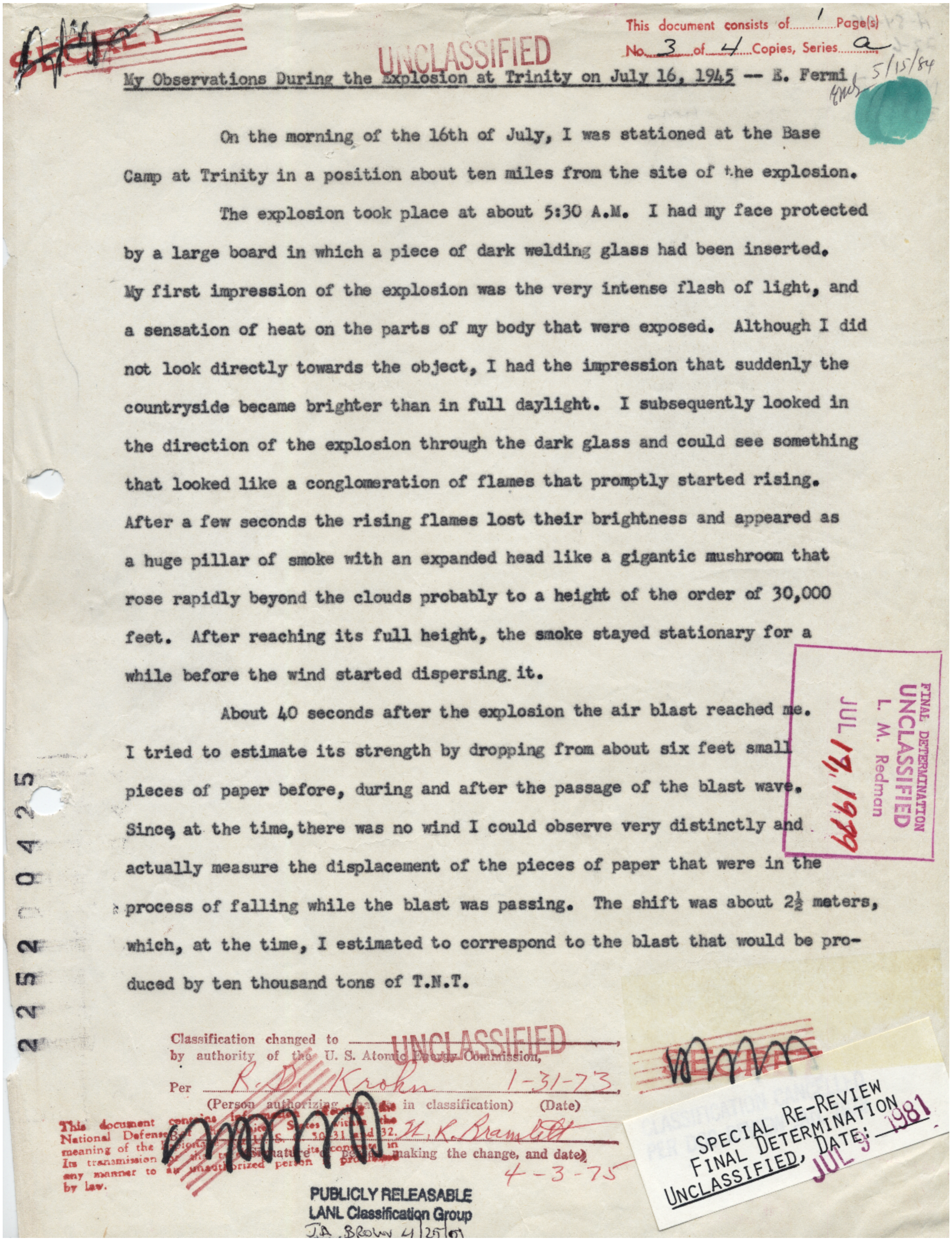
The light scraps of paper that Fermi dropped moved with the air because of their low ballistic coefficients (the ratio of mass to the product of drag coefficient and area projected perpendicular to the velocity through the fluid); they traced the motion of the air in the blast wave. The text of Fermi's memorandum may be found on a number of web sites, and it is reproduced in Fig. 1. This note attempts to reconstruct his reasoning.

Fermi measured the positive phase air displacement D. In the limit of a weak blast wave, D is directly related to the positive phase impulse per unit area $\mathcal{I}$, and measuring D may be the best way of measuring $\mathcal{I}$:

$$D = \int_{v>0} v\, dt = \int_{v>0} \frac{P - P_0}{\rho c_0}\, dt = \frac{\mathcal{I}}{\rho c_0}, \tag{1}$$

where ρc_0 is the acoustic impedance of the air.

What was the understanding of blast wave physics at the time of Trinity? The theory was developed in some detail in the 1944 Los Alamos document LA-165[4], and fitted to empirical data on small (mostly 67 lb) charges in LA-316[5] (June 25, 1945, three weeks before Trinity). Fermi's responsibilities as a nuclear physicist and experimentalist were elsewhere, but he was also a leading theorist and likely familiar with these results. However, the results of[5] do not extend to overpressures below 1 lb/in^2, approximately ten times those at Fermi's observation point, and its results for impulse (directly related to the displacement Fermi measured) for yields $\sim 10,000$ tons relevant to Trinity do not extend to ranges > 2 km, in contrast to the 16 km range to Fermi's observation, and its results for low impulses are obtained for low yields but at the unscaled 100 foot height of burst. Scaling from the May 7, 1945 100 ton high explosive calibration and rehearsal shot[6] might have been possible, but no quantitative


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My Observations During the Explosion at Trinity on July 16, 1945 -- E. Fermi

On the morning of the 16th of July, I was stationed at the Base Camp at Trinity in a position about ten miles from the site of the explosion.

The explosion took place at about 5:30 A.M. I had my face protected by a large board in which a piece of dark welding glass had been inserted. My first impression of the explosion was the very intense flash of light, and a sensation of heat on the parts of my body that were exposed. Although I did not look directly towards the object, I had the impression that suddenly the countryside became brighter than in full daylight. I subsequently looked in the direction of the explosion through the dark glass and could see something that looked like a conglomeration of flames that promptly started rising. After a few seconds the rising flames lost their brightness and appeared as a huge pillar of smoke with an expanded head like a gigantic mushroom that rose rapidly beyond the clouds probably to a height of the order of 30,000 feet. After reaching its full height, the smoke stayed stationary for a while before the wind started dispersing it.

About 40 seconds after the explosion the air blast reached me. I tried to estimate its strength by dropping from about six feet small pieces of paper before, during and after the passage of the blast wave. Since, at the time, there was no wind I could observe very distinctly and actually measure the displacement of the pieces of paper that were in the process of falling while the blast was passing. The shift was about 2½ meters, which, at the time, I estimated to correspond to the blast that would be produced by ten thousand tons of T.N.T.

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FIG. 1. Fermi's original memorandum[3].

data from it are documented in[6]; its purpose appears to have been to test experimental procedure. In addition, the differing specific energies of nuclear and conventional explosives would have made scaling uncertain.

LA-2000[7], dated August 1947, two years after Trinity, presents a comprehensive review edited, and to a substantial extent written, by Hans Bethe, extending the earlier theory and presents the results of numerical calculations apparently performed between 1944 and 1947.[7] contains not only sophisticated analytic theory, as befits its authors, but also in Chap. 6 the results of numerical calculations ("IBM solution, results or run") carried out on the primitive computing machines of the time[8]. Some of those results are used here.

Most of the energy of an airburst is divided into three parts: radiation, "wasted" thermal energy in heated air (partly in the initial fireball and partly in air irreversibly heated by the blast wave) and an outgoing blast wave. A small fraction of the energy appears as radioactive decay and neutron capture gamma-rays. The displacement of the air (and pieces of paper that act as Lagrangian tracers of its motion) far from the explosion includes a net outward displacement that accommodates the increased volume of a bubble of hot gas (whose subsequent buoyant rise creates the famous "mushroom cloud"), and the oscillatory motion of the outgoing blast wave.

II. THE BUBBLE

After pressure equilibrium is achieved, the injection of an energy Y_b into the air increases its volume by

$$\Delta V = \frac{Y_b(\gamma - 1)}{P_0}, \tag{2}$$

where γ is the adiabatic exponent of air and P_0 its initial pressure. At temperatures of less than a few thousand K the vibrational modes of nitrogen and oxygen molecules are not significantly excited and $\gamma \approx 7/5$. At higher temperatures γ is less; full classical excitation of the vibrational modes reduces it to 4/3, and dissociation and endothermic creation of nitric oxide reduce it still further. Most of the volume expansion is attributable to air at lower temperatures, so here we adopt $\gamma = 7/5$.

For an airburst at the top of a 30 m tower[1] and an observer at a distance $\gg$ 30 m, it is possible to approximate the geometry as hemispherical. The effective radius of the hot

bubble, defined as the radius of a hemisphere of volume ΔV,

$$R_H = \left(\frac{3}{2\pi}\frac{Y_b(\gamma-1)}{P_0}\right)^{1/3} \approx \left(\frac{Y_b}{10\text{ kt}}\right)^{1/3} \times 450\,\text{m}, \tag{3}$$

where at the altitude of Trinity of 1.5 km $P_0 \approx 8.5 \times 10^5$ dyne/cm^2. An observer at distance r would see, after all the dynamic motions have settled down but the hot bubble has not yet begun to rise, an outward displacement

$$\Delta r = \frac{\Delta V}{2\pi r^2} = \frac{Y_b}{P_0}\frac{(\gamma-1)}{2\pi r^2} = \frac{R_H^3}{3r^2}. \tag{4}$$

For Fermi's observation at $r = 10$ miles (16 km)

$$\Delta r \approx \left(\frac{Y_b}{10\,\text{kt}}\right) \times 10\,\text{cm}, \tag{5}$$

where 1 kt (kiloton) $= 4.19 \times 10^{19}$ ergs. For modern estimates[2] that Trinity had a yield of 25 ± 2 kt this gives 25 cm, which is much smaller than the 2 1/2 meter displacement he observed, even were there no radiation losses. The explanation of Fermi's observation, and the reconstruction of his argument, must be sought elsewhere.

III. THE BLAST WAVE

The sudden creation of a bubble of hot gas produces an outgoing blast wave. This blast wave is initially very strong, with overpressure $\Delta P \equiv P - P_0 \gg P_0$, where P_0 is the ambient pressure and P the pressure of the shocked and subsequently rarefied air. In the strong shock and point source limit it is described by an analytic theory developed by von Neumann[9], by G. I. Taylor[10] and by Sedov, and described in this issue by Baty and Ramsey[11]. This is not applicable to the weak shock regime in which Fermi made his measurement.

LA-2000[7] also calculates weaker blast waves. However, one crucial element, the fraction of the explosive yield that appears in the weak blast wave ($|\Delta P| \ll P_0$) at large distances, where Fermi measured it, can only be calculated numerically because it involves the coupling of energy through an intermediate strength regime in which no analytic theory is valid. This calculation is described in Chapter 6 of LA-2000[7], but it is difficult to reconcile the results there (Fig. 6) with Fermi's measurement and the known yield[2] of Trinity.

Once the blast wave becomes weak, it is, to a good approximation, a sound wave propagating at the sound speed. The over- and under-pressure ΔP and the fluid speed decay

nearly proportionally to $1/r$ in spherical or hemispherical geometry. The width of the blast wave is nearly constant because the sound speed is nearly its ambient value throughout. Idealized displacement and overpressure profiles are shown in Fig. 2. Unlike the bubble displacement, in the acoustic limit the blast wave displacement returns to zero after the wave passes.

The dependence of shock overpressure on distance from the explosion is shown in Fig. 3 for a spherically symmetric explosion into air at ambient conditions ($P_0 = 1.01 \times 10^6$ dyne/cm^2 and $T = 300$ K). This Figure combines the numerical ("IBM") results with the "semi-acoustic" and acoustic theory for weak and very weak (acoustic) blast waves. §6.3 of LA-2000 states that the calculation was intended to describe a 10 kt explosion but actually described a 13 kt (not including thermal or nuclear radiation) explosion.

For $r \gg R_H$ the leading edge of the blast wave becomes a weak shock ($\Delta P \ll P_0$) that performs very little irreversible work ($\propto (\Delta P)^3$). The blast wave propagates at nearly the speed of sound, $c_0 = 340$ m/s in air at 15 °C, a typical temperature at dawn in July at Trinity (LA-2000 takes $c_0 = 347$ m/s, appropriate to air at 27 °C). The passage of a weak shock produces a velocity discontinuity

$$\Delta v = \frac{\Delta P}{\rho c_0}, \tag{6}$$

defining the acoustic impedance ρc_0, where ρ is the ambient density (very close to the shocked density). The pressure profile of the weak blast wave at one radius is shown in Fig. 4, and the duration of the positive phase ($P > P_0$) is shown in Fig. 5, both for a yield of 13 kt in spherical symmetry.

In this weak blast wave regime the rate of dissipation of blast energy by conversion to thermal "waste energy", so-called because it does not contribute to the destructive effect of the blast, rapidly decreases with increasing r. However, it decreases slowly enough that the energy in the blast wave does not approach a constant asymptote, but rather declines logarithmically with overpressure and distance (Eq. 429 of LA-165[4]).

The energy of a weak hemispherical blast wave whose positive phase has thickness h

$$E_{\text{Blast}} = 2\pi r^2 h \times 2 \times 2 \times \frac{1}{3} \times \frac{\rho(\Delta v)^2}{2}. \tag{7}$$

One factor of 2 results from the contribution of the compression energy, equal to the kinetic energy in an acoustic wave, and another factor of 2 from the contribution of the negative

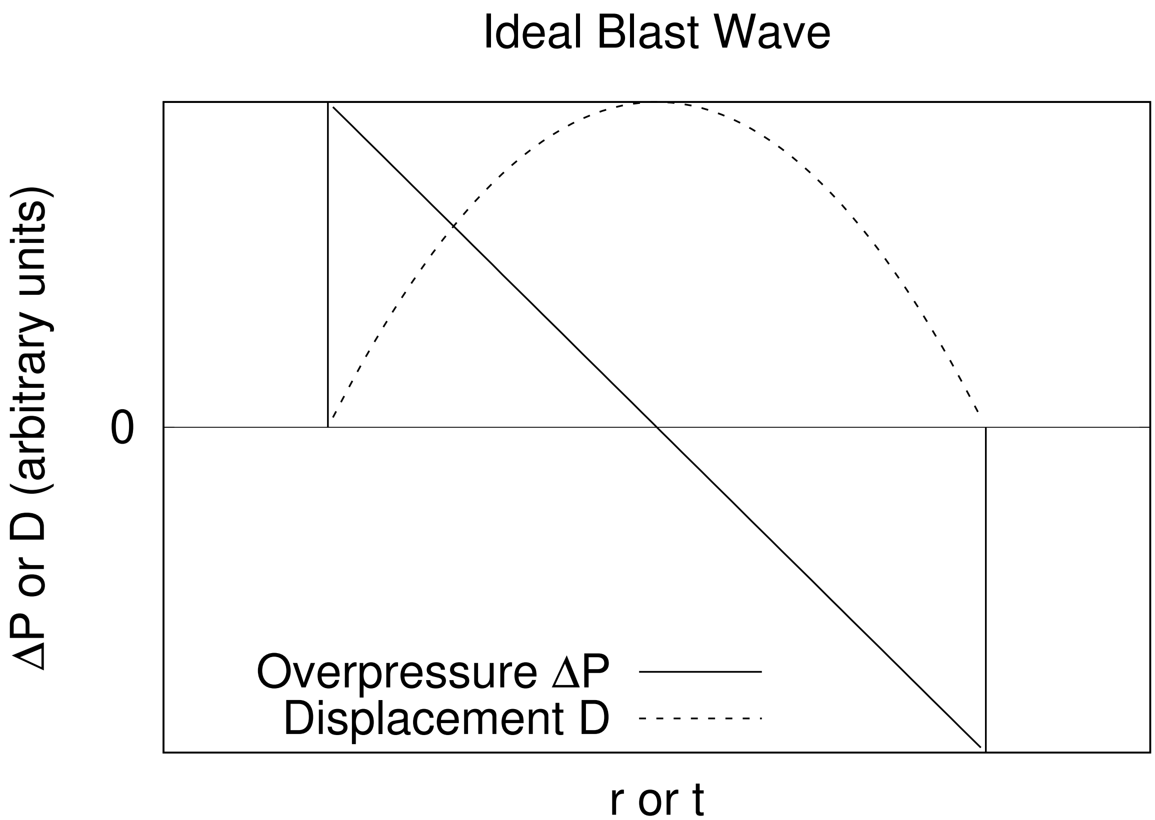


FIG. 2. Idealized blast overpressure and displacement far from the explosion as a function of time at one point or of radius at one instant (if plotted as a function of radius the blast is propagating to the left and the location of the explosion asymptotically distant to the right). This profile is the classic "N-wave"[12], named for its shape; it is produced when an idealized membrane between two regions of infinitesimally differing pressures is abruptly removed. This pressure profile is shown in Fig. 5.1 of LA-2000. The displacement profile is added here, and both were familiar to Fermi. The rarefaction phase of the blast produced by a strong explosion is quite different because there are no rarefaction shocks of finite amplitude; Fig. 4 shows the results of an early calculation from LA-2000. Far from the explosion the blast propagates as an acoustic wave with nearly constant profile, but with amplitude decreasing in proportion to $1/r$.

phase, nominally equal to that of the positive phase (Fig. 2, although Fig. 4 suggests a factor of 1.5 may be more accurate), and the factor of 1/3 comes from averaging v^2 with v varying linearly from 0 to Δv through the blast wave (Fig. 2). The thickness h is comparable to the radius R_H where the shock transitions from strong to weak, scaling $\propto Q^{1/3}$, but grows

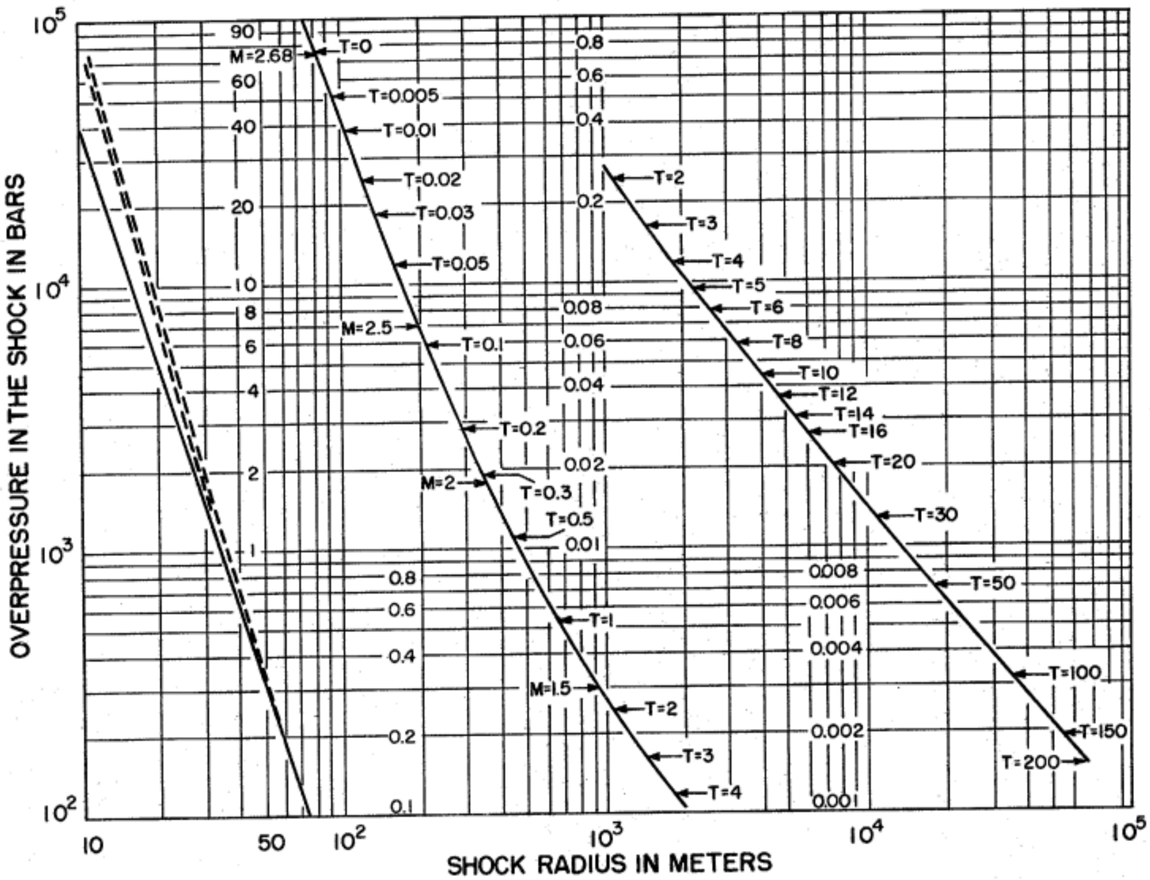


Fig. 6.1 Shock pressure vs distance for nuclear explosion. Time is given in seconds; T = 0 is start of IBM run, 0.012 sec after explosion.

FIG. 3. Calculated shock overpressure *vs.* radius for a 13 kt explosion in ambient air in spherical geometry, corresponding to a 6.5 kt explosion in hemispherical geometry. Note the separate ordinate labels, arrival times T and Mach numbers M for each curve. At Fermi's distance of 16 km, in hemispherical geometry the overpressure would have been 0.008 bar for a 6.5 kt explosion and 0.010 bar for 13 kt. ΔP scales approximately $\propto Q^{1/3}$, where Q is the yield, because the energy density is $\propto (\Delta P)^2$ in an acoustic wave and the blast wave thickness scales $\propto R_H \propto Q^{1/3}$. (Fig. 6.1 of LA-2000)

logarithmically (LA-165[4] and Fig. 5) as the weak blast wave propagates.

Fermi had to estimate $h = c_0 \Delta t$ theoretically because it would not have been easy to measure Δt without photographic or electronic equipment. At $\Delta P = 0.008 P_0$ (16 km from a 13 kt explosion in spherical geometry; Fig. 3) the duration of the positive phase

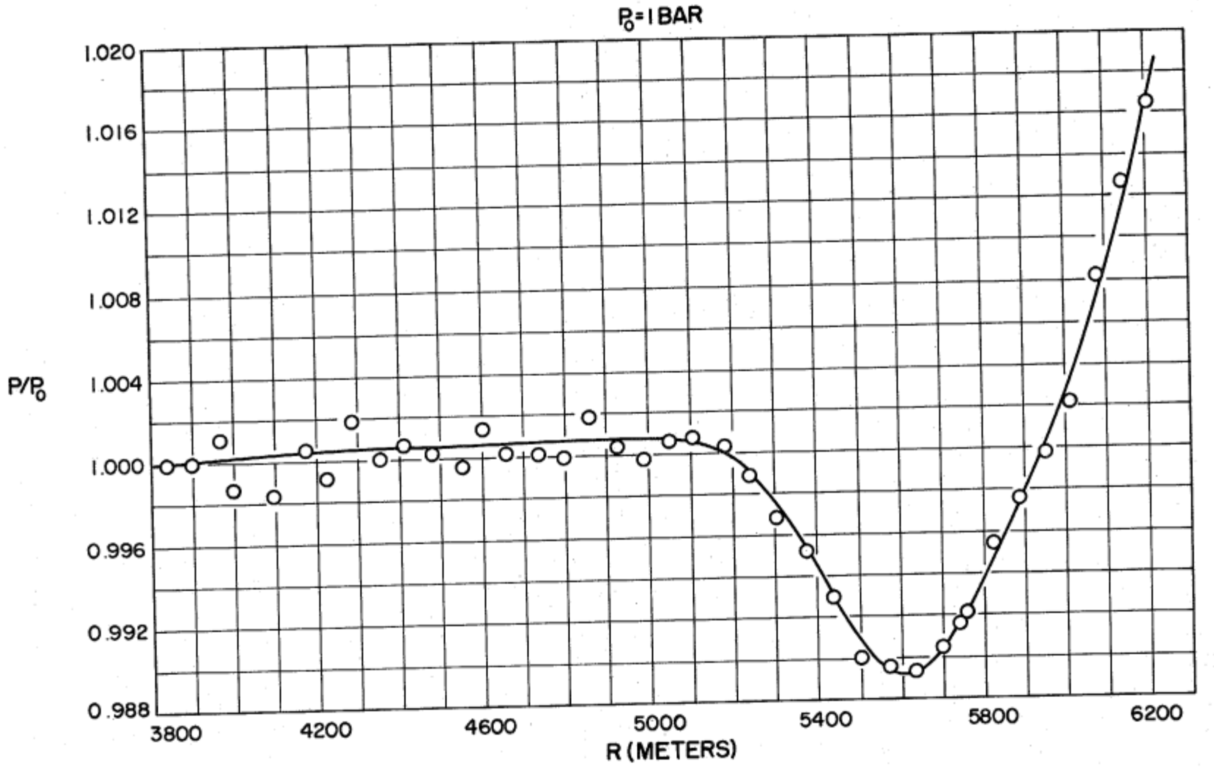


Fig. 6.13 Pressure vs distance at a fixed time

FIG. 4. Calculated blast wave pressure profile far from the explosion for a yield of 13 kt in spherical geometry or 6.5 kt in hemispherical geometry. The width and profile $P(R)$ of the blast wave vary only very slowly from this radius (the greatest radius for which this is shown in LA-2000) outward, while the overpressure ΔP scales $\propto 1/r$. This profile is not close to the ideal profile of Fig. 2, indicating that the model is only approximate. At the shock P/P_0 abruptly rises from unity, but the rise (a vertical line in Fig. 2) is not shown here. According to §6.4 of LA-2000, $\Delta P = 0.0251 P_0$ at 6270 m with a positive phase 290 m wide, but the portion of the pressure curve with $\Delta P > 0.016 P_0$ and $r > 6200\,\text{m}$ is not shown. If the shock is actually at 6270 m, as stated, the positive phase is 320 m wide. The negative phase is much wider. (Fig. 6.13 of LA-2000)

$\Delta t = h/c_0 = 1.14\,\text{s}$ for the same explosion (Fig. 5). Scaling to 13 kt in hemispherical geometry while holding ΔP constant doubles the effective yield and multiplies R_H and Δt by $2^{1/3}$. Then $\Delta t = 1.44\,\text{s}$ and $h = c_0 \Delta t \approx 500\,\text{m}$. This is (probably fortuitously) very close to the $R_H = 490\,\text{m}$ estimated from Eq. 3.

At $\Delta P = 0.02 P_0$, as shown in Fig. 4 at 6 km for 13 kt in spherical geometry, Fig. 5

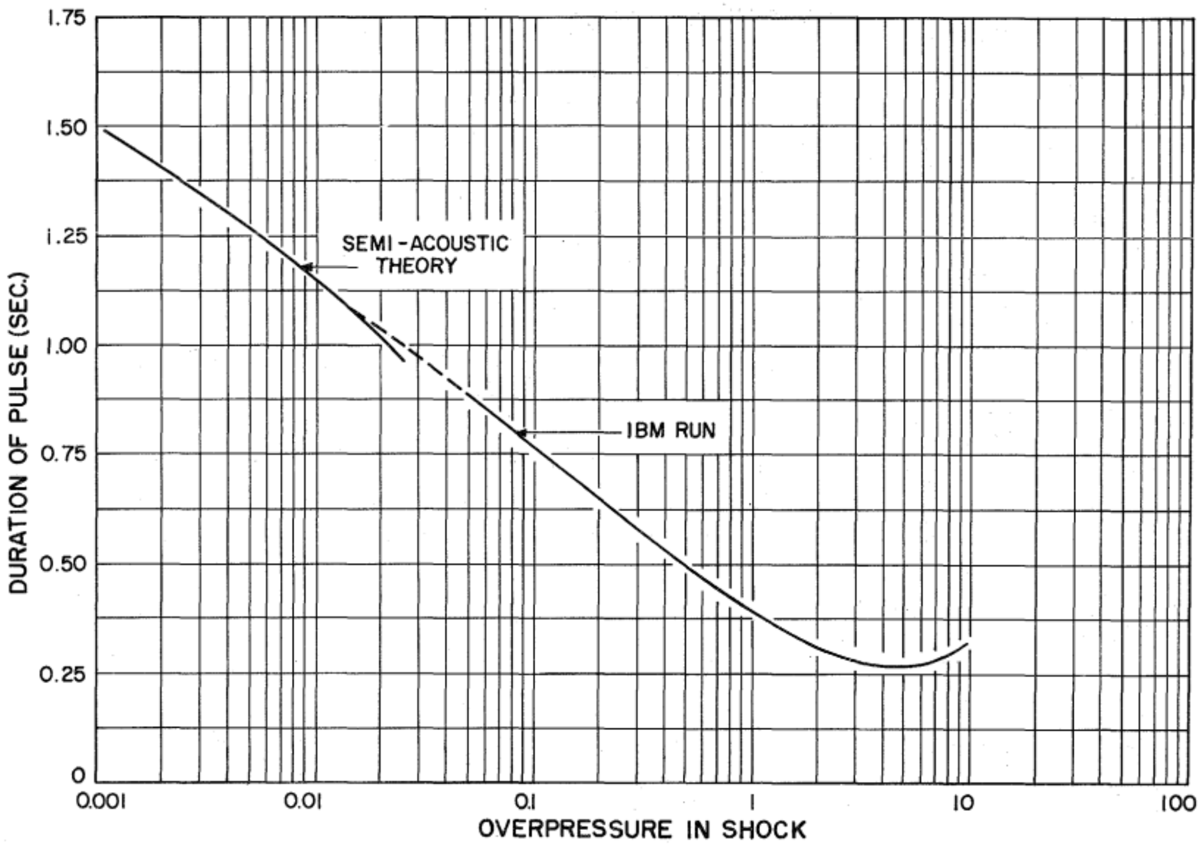


Fig. 6.14 Deviation of the positive phase of a pulse at a fixed distance

FIG. 5. Duration Δt of the positive phase as a function of overpressure (original caption has "Deviation" for "Duration"). The blast wave width increases logarithmically even in the acoustic regime because the overpressure, although small, is not infinitesimal. The "IBM Run" was the full numerical calculation in the moderately strong and strong shock regimes, while the "semi-acoustic" theory of LA-2000 calculates the flow to second order in $\Delta P/P_0$; a first-order calculation is strictly valid only in the acoustic limit. In this limit the positive phase duration Δt and thickness h are related by $h = c_0 \Delta t$. (Fig. 6.14 of LA-2000)

indicates $\Delta t = 1.04\,\text{s}$ and $h = 360\,\text{m}$ rather than the 320 m indicated by Fig. 4 or the 290 m of §6.4 of LA-2000. This should be taken as a measure of the resolution and precision of the calculations.

From the results reported in LA-2000 (or the more approximate elementary analytic result Eq. 3) Fermi had a good idea of the value of h at his observation point for yields in the expected range. Fortunately, Δt and h depend only logarithmically on ΔP (LA-165[4]

and Fig. 5). In principle, he could have used the computed dependence of h on yield to solve interatively for the yield, but this dependence is so weak that such a procedure was unnecessary. Measuring a displacement $D = 2.5\,\mathrm{m}$ provided the peak velocity Δv for the linear pressure and velocity profile of Fig. 2. Then

$$\Delta v = \frac{2Dc_0}{h} = 3.4 \text{ m/s}. \tag{8}$$

This is fairly close to the $v = 2.9\,\mathrm{m/s}$ calculated for $\Delta P = 0.010\,\mathrm{bar}$ (scaling the results of Fig. 3 to hemispherical geometry by multiplying ΔP by $2^{1/3}$) from Eq. 6, taking $\rho = 1.0\,\mathrm{kg/m^3}$ at the 1500 m altitude of Trinity and noting that ΔP scales $\propto E_{\mathrm{Blast}}/r^3$, independent of the air density. Substituting h and Δv in Eq. 7 yields

$$E_{\mathrm{Blast}} = 1.5\,\mathrm{kt}. \tag{9}$$

In order to relate his observation to the explosive yield Fermi had to know what fraction of the yield appears as blast wave energy at an overpressure of about 0.01 bar. If he had relied on the IBM calculation reported in Chap. 6 of LA-2000, shown in Fig. 6, he would have been seriously misled. A small extrapolation from these results to $\Delta P = 0.01\,\mathrm{bar}$ would have led to $E_{\mathrm{Blast}} \approx 0.016Q$, implying an impossible $Q \approx 90\,\mathrm{kt}$, far from both Fermi's estimate and the modern radiochemical value of 25 ± 2 kt[2] for the total yield, that includes thermal and nuclear radiation.

Two factors may contribute to this discrepancy. The effects of numerical viscosity were likely not appreciated when the IBM calculations were done, and the decline of E_{Blast}/Q with increasing range in Fig. 6 may be mostly a numerical artefact. In addition, the "wasted" energy contributes to the pressure driving the blast wave, so it is not really wasted. These hypotheses could be readily tested with a modern calculation.

How did Fermi get it right? He could have readily estimated E_{Blast} from Eqs. 7 and 8 or equivalent forms. He would have needed a value for h, but that could have been taken from the numerical calculations (were they available and known to him in 1945), estimated in advance of Trinity as the shock radius R_H (Eq. 3) where $\Delta P = \mathcal{O}(P)$, or inferred at Trinity from eyeball observation of the duration of the positive phase using his paper Lagrangian tracers. He didn't tell us what he did.

Fermi would also have needed a value for E_{Blast}/Q, an essential parameter that the only possibly available numerical calculation got wrong. He was likely familiar with the Bethe-

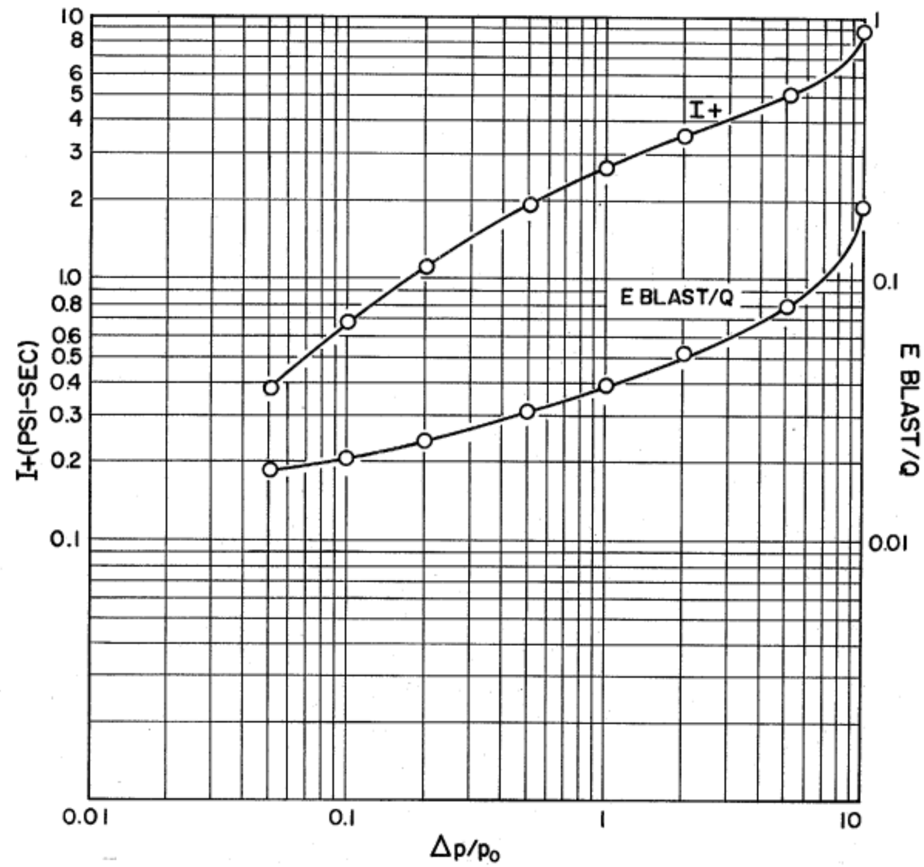


Fig. 6.15 The positive impulse and the energy in the blast. IBM run scaled to 40,000 tons in free air. (Q is the total energy.)

FIG. 6. Impulse of the positive phase of the blast and the fraction of the bomb energy Q that remains in the blast wave. The impulse is scaled to a 40 kt explosion, but the function $E_{\mathrm{Blast}}/Q\,(\Delta P/P_0)$ relating two dimensionless parameters is approximately independent of the size of the explosion (this is only approximate because scaling breaks down for the radiated fraction of Q). (Fig. 6.15 of LA-2000)

Kirkwood-Penney result (Eq. 429 of LA-165[4])

$$\frac{E_{Blast}}{Q} \approx \frac{1}{3\sqrt{\ln P_0/\Delta P}} \approx 0.15, \tag{10}$$

which, combined with his empirical value for E_{Blast} (Eq. 9) immediately leads to his yield estimate $Q \approx 10$ kt.

If the IBM calculations had been done by the time of Trinity, either Fermi was unaware of them or wisely chose to use the analytic theory rather than those apparently quantitative

computational results.

IV. CONCLUSION

At the time Fermi wrote his famous memorandum, the result (Eq. 10) of LA-165[4] was likely well-known at Los Alamos, so that no explanation was necessary. This theory is no longer common knowledge, even though the original documents are readily available to anyone.

Fermi estimated the yield of Trinity to be 10 kt, about 40% of the modern value, from the motion of Lagrangian tracers, his famous scraps of paper. Significant additional energy was radiated, but how much depends on the physics and chemistry of hot air (in particular, the thermodynamics and opacity of oxides of nitrogen) and requires numerical methods to calculate. Fermi's result was therefore a lower bound, and was sufficient to declare Trinity a success.

V. ACKNOWLEDGMENTS

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